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Comments on the gauge models for massless higher spin off-shell supermultiplets

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Abstract

In recent papers arXiv:1103.3564 and arXiv:1103.3565, Gates and Koutrolikos announced the construction of new off-shell formulations for massless higher spin supermultiplets. Here we demonstrate that all of their models are obtained from (some of) those constructed in 1993 by Kuzenko, Postnikov and Sibiryakov by applying special field redefinitions.

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1 Introduction

In four space-time dimensions, the off-shell formulations for massless higher spin $\mathcal{N}=1$ supermultiplets were constructed in [1, 2]. For each superspin $s \geq 1$, half-integer [1] and integer [2], these publications provided two dually equivalent off-shell realizations in $\mathcal{N}=1$ Minkowski superspace. At the component level, each of the two superspin-s actions [1, 2] reduces, *upon* imposing a Wess-Zumino-type gauge and eliminating the auxiliary fields, to a sum of the spin-s and spin-$(s+1/2)$ actions [3].[1] The higher spin superfield theories of [1, 2] were generalized to the case of ant-de Sitter supersymmetry in [5].

Recently, Gates and Koutrolikos [6, 7] have announced the construction of new off-shell formulations for massless higher spin supermultiplets. In the present note we demonstrate that these models are, in fact, not new. They can be obtained from (some of) those constructed in [1, 2] by applying special field redefinitions.

In the remainder of this section, we recall the structure of the constrained superfields used in [1, 2]. Then, in section 2 we consider the case of half-integer superspins. Finally, section 3 is devoted to the case of integer-superspin models.

The off-shell formulations for massless higher spin massless supermultiplets developed in [1, 2] are realized in 4D $\mathcal{N}=1$ Minkowski superspace.[2] They involve the so-called transverse and longitudinal linear superfields, both as dynamical variables and gauge parameters. A complex tensor superfield $\Gamma_{\alpha(k)\dot\alpha(l)}$ is said to be *transverse linear* if it obeys the constraint

$$\bar D^{\dot\beta}\,\Gamma_{\alpha(k)\dot\beta\dot\alpha(l-1)} = 0~, \qquad l>0~. \tag{1.1}$$

A *longitudinal linear* superfield $G_{\alpha(k)\dot\alpha(l)}$ is defined to satisfy the constraint

$$\bar D_{(\dot\alpha_1}\,G_{\alpha(k)\dot\alpha_2\dots\dot\alpha_{l+1})} = 0~. \tag{1.2}$$

The above constraints imply that $\Gamma_{\alpha(k)\dot\alpha(l)}$ and $G_{\alpha(k)\dot\alpha(l)}$ are linear in the usual sense

$$\bar D^2\,\Gamma_{\alpha(k)\dot\alpha(l)} = \bar D^2\,G_{\alpha(k)\dot\alpha(l)} = 0~. \tag{1.3}$$

[1]The results obtained in [1, 2] are reviewed in [4].

[2]Our superspace notation and conventions correspond to [4], in particular the flat superspace covariant derivatives are $D_A = (\partial_a, D_\alpha, \bar D^{\dot\alpha})$. Throughout this paper we consider only Lorentz tensors symmetric in their undotted indices and separately in their dotted ones. For a tensor of type (k,l) with k undotted and l dotted indices we use the shorthand notations $\Psi_{\alpha(k)\dot\alpha(l)} \equiv \Psi_{\alpha_1\dots\alpha_k\dot\alpha_1\dots\dot\alpha_l} = \Psi_{(\alpha_1\dots\alpha_k)(\dot\alpha_1\dots\dot\alpha_l)}$. Quite often we assume that the upper or lower indices, which are denoted by one and the same letter, should be symmetrized, for instance $\phi_{\alpha(k)}\psi_{\alpha(l)} \equiv \phi_{(\alpha_1\dots\alpha_k}\psi_{\alpha_{k+1}\dots\alpha_{k+l})}$. Given two tensors of the same type, their contraction is denoted by $f\cdot g \equiv f^{\alpha(k)\dot\alpha(l)}\,g_{\alpha(k)\dot\alpha(l)} = f^{\alpha_1\dots\alpha_k\dot\alpha_1\dots\dot\alpha_l}\,g_{\alpha_1\dots\alpha_k\dot\alpha_1\dots\dot\alpha_l}$.

In the case $l = 0$, the constraint (1.1) should be replaced by $\bar{D}^2\Gamma_{\alpha(k)} = 0$. The constraint (1.2) for $l = 0$ simply means that $G_{\alpha(k)}$ is chiral, $\bar{D}_{\dot\beta}\, G_{\alpha(k)} = 0$. The constraints (1.1) and (1.2) can be solved in terms of unconstrained prepotentials $\Phi_{\alpha(k)\dot\alpha(l+1)}$ and $\Psi_{\alpha(k)\dot\alpha(l-1)}$ as follows:

$$\Gamma_{\alpha(k)\dot\alpha(l)} = \bar{D}^{\dot\beta}\bar{\Phi}_{\alpha(k)\,\dot\beta\dot\alpha_1\cdots\dot\alpha_l} \ , \tag{1.4a}$$

$$G_{\alpha(k)\dot\alpha(l)} = \bar{D}_{(\dot\alpha_1}\Psi_{\alpha(k)\,\dot\alpha_2\cdots\dot\alpha_l)} \ . \tag{1.4b}$$

The prepotentials are defined modulo gauge transformations of the form:

$$\delta\bar{\Phi}_{\alpha(k)\,\dot\alpha_{(l+1)}} = \bar{D}^{\dot\beta}\bar{\xi}_{\alpha(k)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_{l+1})} \ , \tag{1.5a}$$

$$\delta\Psi_{\alpha(k)\,\dot\alpha_{(l-1)}} = \bar{D}_{(\dot\alpha_1}\zeta_{\alpha(k)\,\dot\alpha_2\cdots\dot\alpha_{l-1})} \ , \tag{1.5b}$$

with the gauge parameters $\bar{\xi}_{\alpha(k)\,\dot\alpha(l+2)}$ and $\zeta_{\alpha(k)\,\dot\alpha(l-2)}$ being unconstrained. In other words, the variations $\delta\bar{\Phi}_{\alpha(k)\,\dot\alpha_{(l+1)}}$ and $\delta\Psi_{\alpha(k)\,\dot\alpha_{l-1})}$ are transverse linear and longitudinal linear, respectively.

2 Half-integer superspin

Two formulations for the massless multiplet of a half-integer superspin $s + 1/2$ (with $s = 1, 2 \ldots$) which were called in Ref. [1] transverse and longitudinal, contain the following dynamical variables respectively:

$$\mathcal{V}^{\perp}_{s+1/2} = \Big\{ H_{\alpha(s)\dot\alpha(s)} \ , \ \Gamma_{\alpha(s-1)\dot\alpha(s-1)} \ , \ \bar{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)} \Big\} \ , \tag{2.1}$$

$$\mathcal{V}^{\parallel}_{s+1/2} = \Big\{ H_{\alpha(s)\dot\alpha(s)} \ , \ G_{\alpha(s-1)\dot\alpha(s-1)} \ , \ \bar{G}_{\alpha(s-1)\dot\alpha(s-1)} \Big\} \ . \tag{2.2}$$

Here $H_{\alpha(s)\dot\alpha(s)}$ is real, $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ transverse linear and $G_{\alpha(s-1)\dot\alpha(s-1)}$ longitudinal linear superfields. The case $s = 1$ corresponds to linearized supergravity (see [4] for a review).

The gauge transformations for the superfields $H_{\alpha(s)\dot\alpha(s)}$, $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ and $G_{\alpha(s-1)\dot\alpha(s-1)}$ postulated in [1] are

$$\delta H_{\alpha(s)\dot\alpha(s)} = g_{\alpha(s)\dot\alpha(s)} + \bar{g}_{\alpha(s)\dot\alpha(s)} \ , \tag{2.3}$$

$$\delta\Gamma_{\alpha(s-1)\dot\alpha(s-1)} = \frac{1}{2}\,\frac{s}{s+1}\,\bar{D}^{\dot\beta}D^{\beta}\bar{g}_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \ , \tag{2.4}$$

$$\delta G_{\alpha(s-1)\dot\alpha(s-1)} = \frac{1}{2}\,\frac{s}{s+1}\,D^{\beta}\bar{D}^{\dot\beta}g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} + \mathrm{i}\,s\,\partial^{\beta\dot\beta}g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \ , \tag{2.5}$$

with the gauge parameter $g_{\alpha(s)\dot\alpha(s)}$ being an arbitrary *longitudinal linear* superfield. It can be seen that $\delta G_{\alpha(s-1)\dot\alpha(s-1)}$ is longitudinal linear.

In the transverse formulation, the action invariant under the gauge transformations (2.3) and (2.4) is

$$
\begin{aligned}
S^{\perp}_{s+1/2} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \, \Big\{ & \frac{1}{8} H^{\alpha(s)\dot\alpha(s)} D^\beta \bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)} \\
& + H^{\alpha(s)\dot\alpha(s)} \big(D_{\alpha_s} \bar D_{\dot\alpha_s} \Gamma_{\alpha(s-1)\dot\alpha(s-1)} - \bar D_{\dot\alpha_s} D_{\alpha_s} \bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)} \big) \\
& + \Big(\bar\Gamma \cdot \Gamma + \frac{s+1}{s} \Gamma \cdot \Gamma \; + \; \text{c.c.} \Big) \Big\} ~.
\end{aligned} \tag{2.6}
$$

In the longitudinal formulation, the action invariant under the gauge transformations (2.3) and (2.5) is

$$
\begin{aligned}
S^{\parallel}_{s+1/2} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \, \Big\{ & \frac{1}{8} H^{\alpha(s)\dot\alpha(s)} D^\beta \bar D^2 D_\beta H_{\alpha(s)\dot\alpha(s)} \\
& - \frac{1}{8} \frac{s}{2s+1} \Big(\big[D_\gamma, \bar D_{\dot\gamma}\big] H^{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)} \Big) \big[D^\beta, \bar D^{\dot\beta}\big] H_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \\
& + \frac{s}{2} \Big(\partial_{\dot\gamma} H^{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)} \Big) \partial^{\beta\dot\beta} H_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \\
& + 2\mathrm{i} \, \frac{s}{2s+1} \, \partial_{\gamma\dot\gamma} H^{\gamma\alpha(s-1)\dot\gamma\dot\alpha(s-1)} \Big(G_{\alpha(s-1)\dot\alpha(s-1)} - \bar G_{\alpha(s-1)\dot\alpha(s-1)} \Big) \\
& + \frac{1}{2s+1} \Big(\bar G \cdot G - \frac{s+1}{s} G \cdot G + \text{c.c.} \Big) \Big\} ~.
\end{aligned} \tag{2.7}
$$

The models (2.6) and (2.7) are dually equivalent [1].

It was pointed out in [5] that there is a natural freedom in the definition of $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$. Specifically, instead of working with $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ one can introduce the following transverse linear superfield

$$
\boldsymbol{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)} := \Gamma_{\alpha(s-1)\dot\alpha(s-1)} + c \, \bar D^{\dot\beta} D^\beta H_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} ~, \tag{2.8}
$$

with c an arbitrary constant. As follows from (2.3) and (2.4), the gauge transformation law of $\boldsymbol{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)}$ is

$$
\delta \boldsymbol{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)} = \Big(c + \frac{1}{2} \frac{s}{s+1} \Big) \bar D^{\dot\beta} D^\beta \bar g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} + c \, \bar D^{\dot\beta} D^\beta g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} ~. \tag{2.9}
$$

Clearly, the transverse theory (2.6) can be re-formulated in terms of $\boldsymbol{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)}$ and its conjugate [5]. However, only in the case $c = 0$, the action functional has the simplest form (2.6).

Now, choosing $c = -\frac{1}{2} \frac{s}{s+1}$ in (2.9) gives

$$
\delta \boldsymbol{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)} = -\frac{1}{2} \frac{s}{s+1} \, \bar D^{\dot\beta} D^\beta g_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} ~. \tag{2.10}
$$

This is exactly the novel transformation law introduced in [6]. More precisely, one has to fill in a couple of technical details in order to see that the transformation (2.10) indeed coincides with that advocated in [6], that is eq. (33) in [6]. First, one has to express $\mathbf{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)}$ in terms of its prepotential, $\bar{\mathbf{\Phi}}_{\alpha(s-1)\,\dot\alpha(s)}$, in accordance with eq. (1.4a), which is defined modulo the *pre*-gauge transformations (1.5a). The point is that the model (32) introduced in [6] is formulated in terms of $H_{\alpha(s)\,\dot\alpha(s)}$, $\bar{\mathbf{\Phi}}_{\alpha(s-1)\,\dot\alpha(s)}$ and its conjugate $\mathbf{\Phi}_{\alpha(s)\,\dot\alpha(s-1)}$. Eq. (2.10) leads to the well-defined gauge transformation of $\bar{\mathbf{\Phi}}_{\alpha(s-1)\,\dot\alpha(s)}$. Secondly, in order to make a direct contact with [6], one should also represent the longitudinal linear parameter $g_{\alpha(s)\dot\alpha(s)}$ in eq. (2.10) in the form $g_{\alpha(s)\dot\alpha(s)} = \bar D_{(\dot\alpha_1} L_{\alpha(s)\,\dot\alpha_2\cdots\dot\alpha_s)}$, for some unconstrained superfield $L_{\alpha(s)\dot\alpha(s-1)}$. As a result, the complete gauge transformation of $\bar{\mathbf{\Phi}}_{\alpha(s-1)\,\dot\alpha(s)}$ is

$$\delta\bar{\mathbf{\Phi}}_{\alpha(s-1)\,\dot\alpha(s)} = -\frac{1}{2}\,\frac{s}{s+1}\,D^\beta\bar D_{(\dot\alpha_1}L_{\beta\alpha(s-1)\dot\alpha_2\cdots\dot\alpha_s)} + \bar D^{\dot\beta}\bar\xi_{\alpha(s)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_s)}\ , \tag{2.11}$$

which is the complex conjugate of the gauge transformation law (33) in [6].

As a consequence of the above discussion, we conclude that the final gauge-invariant action given by Gates and Koutrolikos, eq. (32) in [6], is obtained from (2.6) by applying the field redefinition expressing $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ in terms of $\mathbf{\Gamma}_{\alpha(s-1)\dot\alpha(s-1)}$.

3 Integer superspin

Two formulations of Ref. [2] for the massless multiplet of an integer superspin s (with $s = 1, 2, \ldots$), transverse and longitudinal, contain the following dynamical variables respectively:

$$\mathcal{V}^\perp_s = \Big\{H_{\alpha(s-1)\dot\alpha(s-1)}\ ,\ \Gamma_{\alpha(s)\dot\alpha(s)}\ ,\ \bar\Gamma_{\alpha(s)\dot\alpha(s)}\Big\}\ , \tag{3.1}$$

$$\mathcal{V}^\parallel_s = \Big\{H_{\alpha(s-1)\dot\alpha(s-1)}\ ,\ G_{\alpha(s)\dot\alpha(s)}\ ,\ \bar G_{\alpha(s)\dot\alpha(s)}\Big\}\ . \tag{3.2}$$

Here $H_{\alpha(s-1)\dot\alpha(s-1)}$ is real, $\Gamma_{\alpha(s)\dot\alpha(s)}$ transverse linear and $G_{\alpha(s)\dot\alpha(s)}$ longitudinal linear tensor superfields. The case $s = 1$ corresponds to the gravitino multiplet (see [4] for a review).

The gauge transformations for the superfields $H_{\alpha(s-1)\dot\alpha(s-1)}$, $G_{\alpha(s)\dot\alpha(s)}$ and $\Gamma_{\alpha(s)\dot\alpha(s)}$ postulated in [2] are

$$\delta H_{\alpha(s-1)\dot\alpha(s-1)} = \gamma_{\alpha(s-1)\dot\alpha(s-1)} + \bar\gamma_{\alpha(s-1)\dot\alpha(s-1)}\ , \tag{3.3}$$

$$\delta\Gamma_{\alpha(s)\dot\alpha(s)} = \frac{1}{2}D_{(\alpha_s}\bar D_{(\dot\alpha_s}\gamma_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})} - \mathrm{i}\,s\,\partial_{(\alpha_s(\dot\alpha_s}\gamma_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})}\ , \tag{3.4}$$

$$\delta G_{\alpha(s)\dot\alpha(s)} = \frac{1}{2}\bar D_{(\dot\alpha_s}D_{(\alpha_s}\bar\gamma_{\alpha_1\ldots\alpha_{s-1})\dot\alpha_1\ldots\dot\alpha_{s-1})}\ , \tag{3.5}$$

with the gauge parameter $\gamma_{\alpha(s-1)\dot\alpha(s-1)}$ being an arbitrary *transverse linear* superfield. It can be seen that $\delta\Gamma_{\alpha(s)\dot\alpha(s)}$ is transverse linear.

In the transverse formulation, the action invariant under the gauge transformations (3.3) and (3.4) is as follows:

$$
\begin{aligned}
S_s^{\perp} = -\Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \,\Big\{ & -\frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{1}{8}\frac{s^2}{(s+1)(2s+1)} \Big(\big[D^{\alpha_s}, \bar D^{\dot\alpha_s}\big] H^{\alpha(s-1)\dot\alpha(s-1)}\Big) \big[D_{(\alpha_s}, \bar D_{(\dot\alpha_s}\big] H_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\alpha_{s-1})} \\
& + \frac{1}{2}\frac{s^2}{s+1}\Big(\partial^{\alpha_s\dot\alpha_s} H^{\alpha(s-1)\dot\alpha(s-1)}\Big)\partial_{(\alpha_s(\dot\alpha_s} H_{\alpha_1\dots\alpha_{s-1})\,\dot\alpha_1\dots\dot\alpha_{s-1})} \\
& + 2\mathrm{i}\,\frac{s}{2s+1}\, H^{\alpha(s-1)\dot\alpha(s-1)}\partial^{\alpha_s\dot\alpha_s}\Big(\Gamma_{\alpha(s)\dot\alpha(s)} - \bar\Gamma_{\alpha(s)\dot\alpha(s)}\Big) \\
& + \frac{1}{2s+1}\Big(\bar\Gamma\cdot\Gamma - \frac{s+1}{s}\Gamma\cdot\Gamma + \mathrm{c.c.}\Big)\Big\}\,.
\end{aligned} \tag{3.6}
$$

In the longitudinal formulation, the action invariant under the gauge transformations (3.3) and (3.5) is

$$
\begin{aligned}
S_s^{\parallel} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^8 z \,\Big\{ & \frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} D^\beta \bar D^2 D_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)}\Big(D^\beta \bar D^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar D^{\dot\beta} D^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\Big) \\
& + \Big(\bar G\cdot G + \frac{s}{s+1}\, G\cdot G + \mathrm{c.c.}\Big)\Big\}\,.
\end{aligned} \tag{3.7}
$$

The models (3.7) and (3.6) are dually equivalent [2].

It was pointed out in [5] that there is a natural freedom in the definition of $G_{\alpha(s)\dot\alpha(s)}$. Specifically, instead of working with $G_{\alpha(s)\dot\alpha(s)}$ one can introduce the following longitudinal linear superfield

$$
\boldsymbol{G}_{\alpha(s)\dot\alpha(s)} := G_{\alpha(s)\dot\alpha(s)} + c\,\bar D_{(\dot\alpha_s} D_{(\alpha_s} H_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\dot\alpha_{s-1})}\ , \tag{3.8}
$$

with c an arbitrary constant. In accordance with (3.3) and (3.5), the gauge transformation law of $\boldsymbol{G}_{\alpha(s)\dot\alpha(s)}$ is

$$
\delta\boldsymbol{G}_{\alpha(s)\dot\alpha(s)} = c\bar D_{(\dot\alpha_s} D_{(\alpha_s}\gamma_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\dot\alpha_{s-1})} + (c+\frac{1}{2})\bar D_{(\dot\alpha_s} D_{(\alpha_s}\bar\gamma_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\dot\alpha_{s-1})}\ . \tag{3.9}
$$

Clearly, the longitudinal theory (3.7) can be re-formulated in terms of $\boldsymbol{G}_{\alpha(s)\dot\alpha(s)}$ and its conjugate [5]. However, only in the case $c=0$, the action functional has the simplest form (3.7).

Now, choosing $c = -1/2$ in (3.9) gives

$$\delta \boldsymbol{G}_{\alpha(s)\dot\alpha(s)} = -\frac{1}{2}\bar D_{(\dot\alpha_s} D_{(\alpha_s}\, \gamma_{\alpha_1\dots\alpha_{s-1})\dot\alpha_1\dots\dot\alpha_{s-1})} ~. \tag{3.10}$$

This is exactly the novel transformation law introduced in [7]. More precisely, one has to fill in several technical details in order to see that the transformation (3.10) indeed coincides with that advocated in [7], that is eq. (27) in [7]. First, one has to express $\boldsymbol{G}_{\alpha(s)\dot\alpha(s)}$ in terms of its prepotential, $\boldsymbol{\Psi}_{\alpha(s)\,\dot\alpha(s-1)}$, in accordance with eq. (1.4b), which is defined modulo the *pre*-gauge transformations (1.5b). The point is that the model (37) introduced in [7] is formulated in terms of $H_{\alpha(s-1)\,\dot\alpha(s-1)}$, $\boldsymbol{\Psi}_{\alpha(s)\,\dot\alpha(s-1)}$ and its conjugate $\bar{\boldsymbol{\Psi}}_{\alpha(s-1)\,\dot\alpha(s)}$. Eq. (3.10) leads to the well-defined gauge transformation of $\boldsymbol{\Psi}_{\alpha(s)\,\dot\alpha(s-1)}$. Secondly, in order to make a direct contact with [6], one should also represent the transverse linear parameter $\gamma_{\alpha(s-1)\dot\alpha(s-1)}$ in eq. (3.10) in the form $\gamma_{\alpha(s-1)\dot\alpha(s-1)} = \bar D^{\dot\beta} \bar L_{\alpha(s-1)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_{s-1})}$, for some unconstrained superfield $\bar L_{\alpha(s-1)\dot\alpha(s)}$. As a result, the complete gauge transformation of $\boldsymbol{\Psi}_{\alpha(s)\,\dot\alpha(s-1)}$ is

$$\delta \boldsymbol{\Psi}_{\alpha(s)\,\dot\alpha(s-1)} = -\frac{1}{2} D_{(\alpha_1}\, \bar D^{\dot\beta} \bar L_{\alpha_2\cdots\alpha_s)\,(\beta\dot\alpha_1\cdots\dot\alpha_{s-1})} + \bar D_{(\dot\alpha_1} \zeta_{\alpha(s)\,\dot\alpha_2\cdots\dot\alpha_{s-1})} ~, \tag{3.11}$$

which is the gauge transformation law (27) in [7].

As a consequence of the above discussion, we conclude that the final gauge-invariant action given by Gates and Koutrolikos, eq. (37) in [7], is obtained from our action (3.7) by applying the field redefinition expressing $G_{\alpha(s)\dot\alpha(s)}$ in terms of $\boldsymbol{G}_{\alpha(s)\dot\alpha(s)}$.

It should be mentioned that Refs. [6, 7] presented interesting reformulations of the models studied, which involve an auxiliary unconstrained real superfield $B_{\alpha(s-1)\dot\alpha(s-1)}$. These reformulations may be useful and deserve further studies.

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